COMPLEX NUMBERS

A number of the form a+ ib, where a and b are real numbers and i = $\sqrt{-1}$, is known as complex number, so

$$C = \{z = a + ib;\ \text{a and b are real numbes and } i = \sqrt{-1}\}$$

Here, ‘a’ is called real part of z and ‘b’ is called imaginary part of z.

Powers of iota:

$$i = \sqrt{-1},\ i^2 = \sqrt{-1} \times \sqrt{-1} = -1,\ \ i^3 = i^2 \times i = (-1) \times i = -i,$$
$$i^4 = i^2 \times i^2 = (-1) \times (-1) = 1$$

In general, we have

$$i^{4k} = 1,\ \ i^{4k+1} = i,\ i^{4k+2} = -1,\ i^{4k+3} = -i$$

Algebra of Complex Numbers:

If $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$ are any two complex numbers, then

(i) Addition of complex numbers:

$$\begin{aligned} z_1 + z_2 &= (a_1 + ib_1) + (a_2 + ib_2) \\ &= (a_1 + a_2) + i(b_1 + b_2) \end{aligned}$$

(ii) Difference of complex numbers:

$$\begin{aligned} z_1 - z_2 &= (a_1 + ib_1) - (a_2 + ib_2) \\ &= (a_1 - a_2) + i(b_1 - b_2) \end{aligned}$$

(iii) Multiplication of complex numbers:

$$\begin{aligned} z_1 z_2 &= (a_1 + ib_1)(a_2 + ib_2) \\ &= (a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1) \end{aligned}$$

Modulus of Complex number:

t z = a + ib be a complex number. Then modulus of z, denoted as $|z|$, is given by $|z| = \sqrt{a^2 + b^2}$

Properties of Modulus:

(i) $|z_1 z_2| = |z_1||z_2|$

(ii) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}, |z_2| \neq 0$

Conjugate of a complex number:

Let z = a + ib be a complex number. Then conjugate of z, denoted as $\bar{z}$, is given by $\bar{z} = a - ib$

Properties of Conjugate:

(i) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$

(ii) $\overline{z_1 \pm z_2} = \overline{z_1} \pm \overline{z_2}$

(iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}; z_2 \neq 0$

Multiplicative Inverse of a Complex number:

Let z = a + ib be a complex number. Then multiplicative inverse of z, denoted as z^{-1}, is given by

$$z^{-1} = \frac{1}{z} = \frac{1}{a+ib} = \frac{1}{a+ib} \times \frac{a-ib}{a-ib} = \frac{a-ib}{a^2+b^2} = \frac{\bar{z}}{|z|^2}$$

Square roots of a complex number:

Let z = a + ib be a complex number. Then square root of z, is given by $\sqrt{z} = \left(\frac{\sqrt{a^2+b^2}+a}{2}\right)^{\frac{1}{2}} \pm i\left(\frac{\sqrt{a^2+b^2}-a}{2}\right)^{\frac{1}{2}}$

Polar form of a Complex number:

The polar form of a complex number z = a + ib is given by

$z = r(\cos\theta + i\sin\theta)$, where $r = \sqrt{a^2 + b^2}$, $\cos\theta = \frac{a}{r}$ and $\sin\theta = \frac{b}{r}$.

Here, θ is known as argument of z and is denoted as (arg z).

The value of θ, such that $-\pi < \theta \le \pi$, is called the principal argument of z and is denoted as (Arg z).