# BINOMIAL THEOREM:

## Binomial theorem:

The expansion of $(a+b)^n$ for any positive integer n is given by Binomial Theorem, which is

$$(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b^1 + {}^nC_2a^{n-2}b^2 + \ldots\ldots.. + {}^nC_{n-1}a^1b^{n-1} + {}^nC_nb^n$$

## Useful Terms:

1. **Binomial Coefficients:** The coefficients ${}^nC_r$ occurring in the Binomial Theorem are called Binomial coefficients.
2. **Pascal's triangle:** The binomial coefficients of the expansions are arranged in an array. This array is called Pascal's triangle.

| Index | Coefficients |
|---|---|
| 0 | 1 |
| 1 | 1 1 |
| 2 | 1 2 1 |
| 3 | 1 3 3 1 |
| 4 | 1 4 6 4 1 |

3. **General Term:** The general term of an expansion $(a+b)^n$ is $T_{r+1} = {}^nC_ra^{n-r}b^r$.

4. **Middle term/terms in the expansion of** $(a + b)^n$:

   (i) If n is even, then the given expansion has only one middle term which is given by $\left(\frac{n}{2}+1\right)^{th}$ term.

   (ii) If n is odd, then the given expansion has two middle terms, namely $\left(\frac{n+1}{2}\right)^{th}$ term and $\left(\frac{n+1}{2}+1\right)^{th}$ term.

5. **The $r^{th}$ term from the end** :

$r^{th}$ term from the end of $(a+b)^n$ = $r^{th}$ term from the beginning of $(b+a)^n$.

6. **Properties of Binomial Coefficients:**

**i.** Sum of the binomial coefficients is equal to $2^n$

$${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots\ldots\ldots\ldots + {}^nC_n = 2^n$$

**ii.** The sum of all the odd binomial coefficients is equal to the sum of all the even binomial coefficients and each is equal to $2^{n-1}$.

$${}^nC_0 + {}^nC_2 + {}^nC_4 + {}^nC_6 \ldots\ldots\ldots = {}^nC_1 + {}^nC_3 + {}^nC_5 + {}^nC_7 \ldots\ldots\ldots = 2^{n-1}$$